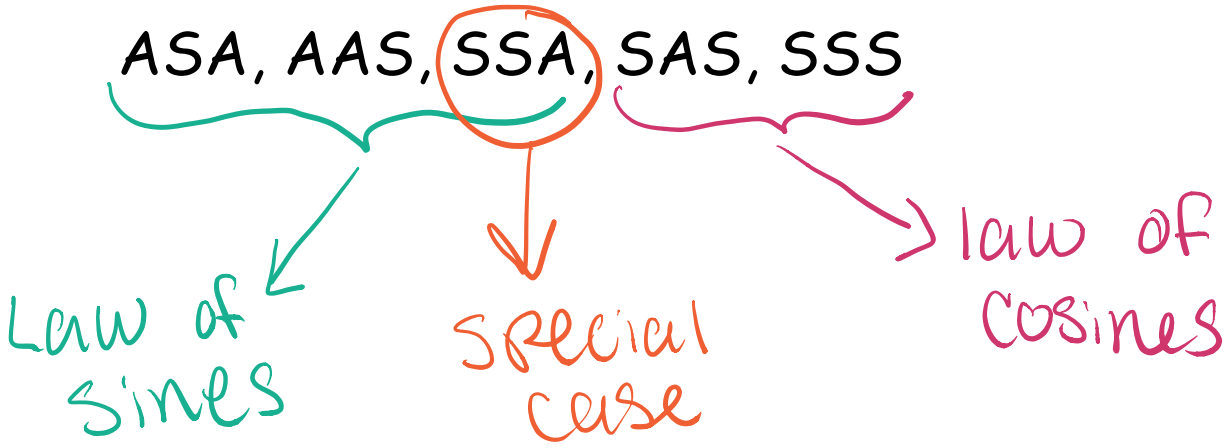
Unit 9 Day 11:
Trigonometry
(9.7) Law of Sines

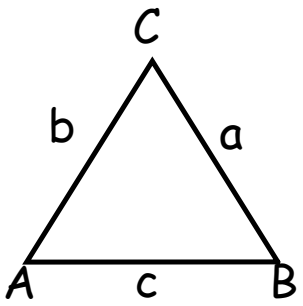
In order to solve a non-right triangle you need to know one side and 2 other pieces of info.



Law of Sines

If triangle ABC has sides of length a, b, and c, then:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



* NOTE: Capital letters denote angles

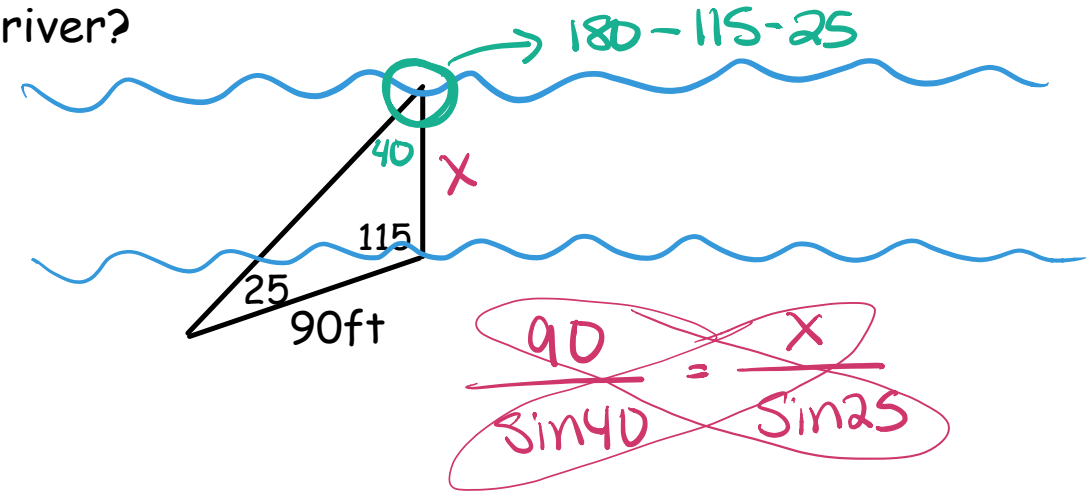
lower case letters denote sides.

Very important to know this!

ROUNDING: Sides = 1 decimal places
Angles = Nearest whole #

Example

A surveyor wants to find the width of a river from a particular point on the shortline for construction of a new bridge. The surveyor's measurements are shown. How wide is the river?



$$\frac{90}{\sin 40} = \frac{X}{\sin 25}$$

$$X \sin 40 = 90 \sin 25$$

$$X = \frac{90 \sin 25}{\sin 40}$$

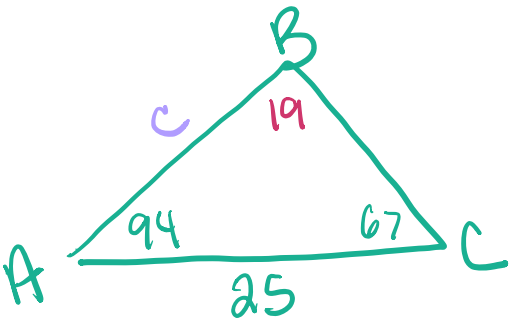
$$X = 59.2 \text{ ft}$$

Example

Find the length c, given that A=94, C=67, and b=25.

$180 - 94 - 67 = B$

$19 = B$



$$\frac{25}{\sin 19} = \frac{c}{\sin 67}$$

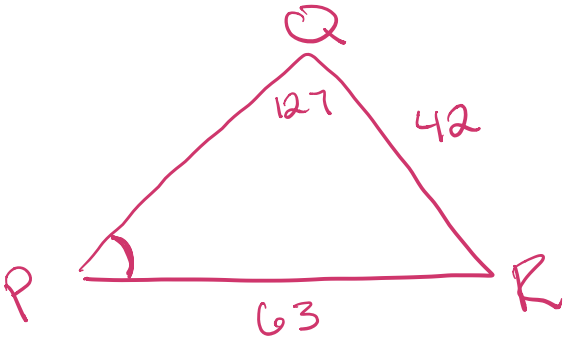
$$c \sin 19 = 25 \sin 67$$

$$c = \frac{25 \sin 67}{\sin 19}$$

$$c = 70.7$$

Example

Find the measure of angle P given that Q=127, q=63, and p=42.



$$\frac{42}{\sin P} = \frac{63}{\sin 127}$$

$$63\sin P = 42\sin 127$$

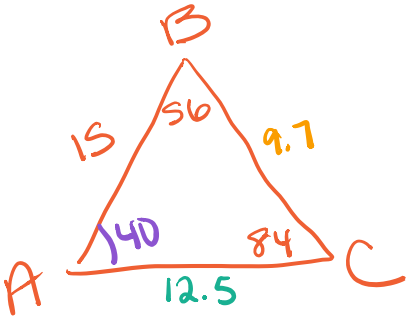
$$\sin P = \frac{42\sin 127}{63}$$

$$P = \sin^{-1}\left(\frac{42\sin 127}{63}\right)$$

$$P = 32^{\circ}$$

Example

Sketch ABC, then find all angle measures and side lengths. B=56, C=84, c=15



$A = 180-84-56$

$A = 40$

$\frac{15}{\sin 84} = \frac{b}{\sin 56}$

$b\sin 84 = 15\sin 56$

$b = \frac{15\sin 56}{\sin 84}$

$b = 12.5$

$\frac{15}{\sin 84} = \frac{a}{\sin 40}$

$a\sin 84 = 15\sin 40$

$a = \frac{15\sin 40}{\sin 84}$

$a = 9.7$

Remember how I said SSA was a special case?

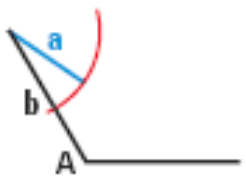
With this given info. you can have 1 triangle , 2 triangles or no triangle at all.

This only is a possibility if one of the sides that is given is the same letter as the angle that is given.

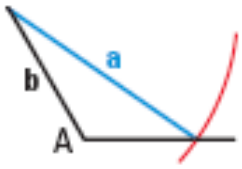
Possible Triangles in the SSA Case

Consider a triangle in which you are given a, b, and A. By fixing side b and angle A, you can sketch the possible positions of side a to figure out how many triangles can be formed. In the diagrams below, note that $h = b \sin A$.

A is obtuse.

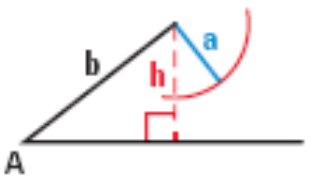


$a \leq b$
No triangle

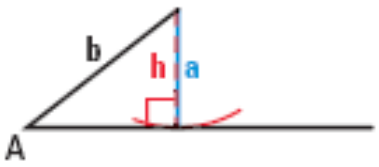


$a > b$
One triangle

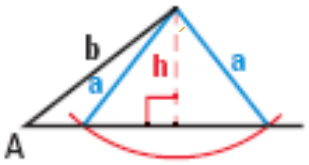
A is acute.



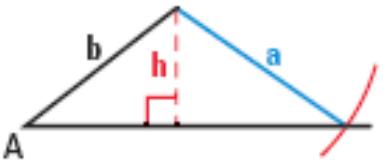
$h > a$
No triangle



$h = a$
One triangle



$h < a < b$
Two triangles



$a > b$
One triangle

Steps to determine # of triangles.

1. Is it SSA?
- if yes, continue
-if no, one triangle

2. Is the side across from the given angle shorter than the second side?
- if yes, continue
-if no, one triangle

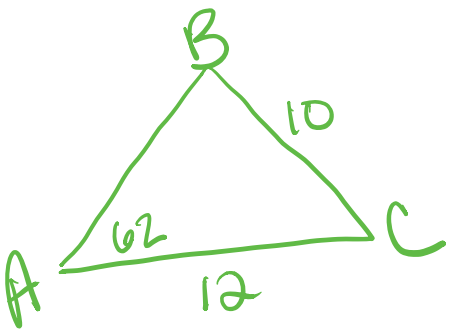
3. Find the height h=xsiny
where y=given angle
x=second side

4. Is the side across from the given angle greater than h?
-if yes, 2 triangles
-if no , no triangle

Example

Determine if there is one triangle, two triangles or no triangle.

$A=62, a=10, b=12$



① is it SSA? yes ✓

② is $a < b$? yes ✓ $10 < 12$

③ Find height $h = x \sin y$

$h = 12 \sin 62$

$h = 10.6$

④ is $a > h$? NO

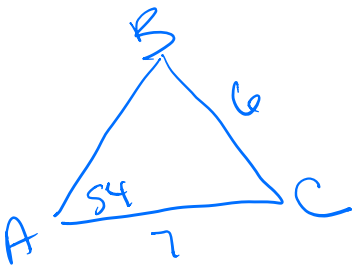
$10 \not> 10.6$

Conclusion: NO △

Example

Determine if there is one triangle, two triangles or no triangle.

A=54, a=6, b=7



① is it SSA? yes ✓

② is $a < 7$? yes ✓ $6 < 7$

③ find height

$h = 7\sin 54$

$h = 5.7$

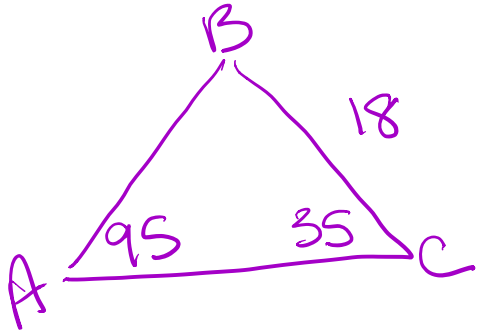
④ is $a > h$? yes ✓ $6 > 5.7$

Conclusion: 2 △'s

Example

Determine if there is one triangle, two triangles or no triangle.

A=95, a=18, C=35



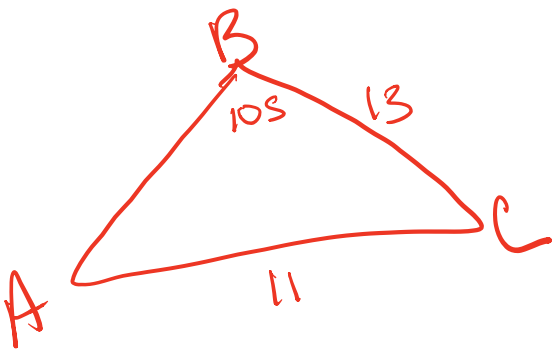
① is it SSA? NO

conclusion: one △

Example

Determine if there is one triangle, two triangles or no triangle.

B=105, b=11, a=13



① is it SSA? yes ✓

② is $b < a$? yes ✓ $11 < 13$

③ $h = 13\sin 105$

$h = 12.6$

④ is $b > h$? NO $11 \not> 12.6$

Conclusion: NO △

HW:
13.5 Law of Sines WS

Mini Quiz:
Wednesday 3/11 Thursday 3/12